# The Crystal Structure of Ytterbium Metaphosphate, $\mathbf{Y b P}_{3} \mathbf{O}_{9}{ }^{*}$ 

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#### Abstract

A single-crystal X-ray diffraction analysis has been performed on $\mathrm{YbP}_{3} \mathrm{O}_{9}$ synthesized by a flux method. The structure is monoclinic, with space group $P 2_{1} / c, Z=12$, and cell parameters $a=11 \cdot 219$ (2), $b=$ 19.983 (3), $c=9.999$ (3) $\AA, \beta=97.30$ (2) ${ }^{\circ}$. A full-matrix least-squares refinement gave $R=0.076, R_{w}=$ 0.064 for 1472 independent reflections. Oxygen atoms form a tetrahedron around each $P$ atom and an octahedron around each Yb atom. The $\mathrm{PO}_{4}$ tetrahedra share corners to form helical ribbons running along the $c$ axis. The $\mathrm{YbO}_{6}$ octahedra are isolated from each other, with no O atom common to any two Yb atoms. The shortest $\mathrm{Yb}-\mathrm{Yb}$ distance is $5 \cdot 610 \AA$, and the concentration of Yb atoms is $5.40 \times 10^{21}$ $\mathrm{cm}^{-3} . \mathrm{Y}_{1-x} \mathrm{Nd}_{x} \mathrm{P}_{3} \mathrm{O}_{9}$ solid solutions appear to be promising materials for efficient $\mathrm{Nd}^{3+}$ lasers.


## Introduction

Three types of rare-earth phosphates have been reported: orthophosphates $\left(\mathrm{LnPO}_{4}\right)$, metaphosphates $\left(\mathrm{LnP}_{3} \mathrm{O}_{9}\right)$, and ultraphosphates $\left(\mathrm{LnP}_{5} \mathrm{O}_{14}\right)$. The basic structure of the phosphate moiety is determined by its $\mathrm{O} / \mathrm{P}$ ratio.
The O atoms in phosphates are all bonded tetrahedrally to P atoms. The $\mathrm{PO}_{4}$ tetrahedra can be isolated from each other or share corners, but edge- and facesharing are never observed, probably because of the repulsion between $\mathrm{P}^{5+}$ ions. Therefore in metaphosphates, with $\mathrm{O} / \mathrm{P}=3$, all the tetrahedra share corners to form one-dimensional chains. As the $\mathrm{O} / \mathrm{P}$ ratio increases above the critical value of 3 , the chains are increasingly broken, until in orthophosphates ( $\mathrm{O} / \mathrm{P}=4$ ) the tetrahedra are all isolated. For O/P ratios of less than 3 , as in the ultraphosphates ( $\mathrm{O} / \mathrm{P}=2 \cdot 8$ ), the chains become cross-linked. However a three-dimensional network cannot be formed. Such a network would require each $\mathrm{PO}_{4}$ tetrahedron to share corners with 4 others for an $\mathrm{O} / \mathrm{P}$ ratio of 2 , but the lower limit of this ratio is $2 \cdot 5$, as in $\mathrm{P}_{2} \mathrm{O}_{5}$.
In the orthophosphates the $\mathrm{PO}_{4}$ tetrahedra are isolated from each other, while in the other compounds they share corners to form ribbons and cross-linked chains, respectively. Since the corner-sharing tetrahedra are flexible, their detailed arrangement depends on the size of the rare-earth atom, and therefore the larger rare earths (from La to Gd or Tb ) form structures differently from the smaller ones (from Gd or Tb to Lu , plus Y). The orthophosphates with large Ln atoms are dimorphic-hexagonal or monoclinic (Mooney, 1948), while those with small Ln atoms are tetragonal (Schwarz, 1963). In the metaphosphate series, large Ln atoms form an orthorhombic structure (Hong, 1974), and small Ln atoms form a monoclinic structure that will be described in this paper. The ultraphosphates

[^0]with large Ln atoms are monoclinic with pairs of crosslinked $\mathrm{PO}_{4}$ chains (Hong, 1974); those with small Ln atoms have a different monoclinic structure in which all the chains in each two-dimensional layer are crosslinked (Hong \& Pierce, 1974). An orthorhombic structure with pairs of cross-linked chains is formed by atoms between Dy and Er (Beucher, 1970; Durif, 1971).

We have measured a fluorescence lifetime of $490 \mu \mathrm{~s}$ for the ${ }^{4} \mathrm{~F}_{3 / 2}$ level of $\mathrm{Nd}^{3+}$ in $\mathrm{Y}_{0.9} \mathrm{Nd}_{0.1} \mathrm{P}_{3} \mathrm{O}_{9}$, which is to be compared with $160 \mu \mathrm{~s}$ in $\mathrm{La}_{0.9} \mathrm{Nd}_{0.1} \mathrm{P}_{3} \mathrm{O}_{9}$ and $300 \mu \mathrm{~s}$ in $\mathrm{La}_{0.9} \mathrm{Nd}_{0.1} \mathrm{P}_{5} \mathrm{O}_{14}$ (Dwight, Hong \& Pierce, 1973). In order to find an explanation for this unusually long lifetime we have used the single-crystal X-ray diffraction method to determine the structure of $\mathrm{YbP}_{3} \mathrm{O}_{9}$, which is isostructural with $\mathrm{YP}_{3} \mathrm{O}_{9}$.

## Experimental procedure

Small single crystals of $\mathrm{YbP}_{3} \mathrm{O}_{9}$ were grown from a flux of $96 \%$ reagent-grade $\mathrm{H}_{3} \mathrm{PO}_{4}$ and $4 \% \mathrm{NaF}$ by weight. A mixture of $\mathrm{Yb}_{2} \mathrm{O}_{3}$ and $\mathrm{P}_{2} \mathrm{O}_{5}$ in 1 to 5 molar ratio was combined with about half its weight of flux and preheated in a Pt crucible to $500^{\circ} \mathrm{C}$ for 4 h , held at $950^{\circ} \mathrm{C}$ for 15 h , cooled at $5^{\circ} \mathrm{C} \mathrm{h}^{-1}$ to $600^{\circ} \mathrm{C}$, and quenched. After being washed with hot water to dissolve any excess phosphoric acid, the product was found to include a considerable number of irregular transparent crystals with dimensions of about 0.05 to 0.2 mm .

For X-ray diffraction studies, a small crystal of about $0.1 \times 0.1 \times 0.1 \mathrm{~mm}$ was mounted on a goniometer head about the $b$ axis. Oscillation and Weissenberg photographs showed diffraction symmetry $2 / \mathrm{m}$. The systematic absences were $0 k 0, k=2 n+1$ and $h 0 l, l=2 n+1$, which indicate uniquely the monoclinic space group $P 2_{1} / c$ (No. 14). Values of 20 were measured for 20 reflections carefully centered on the $K \alpha_{1}$ and $K \alpha_{2}$ components of Mo radiation ( $\lambda_{1}=0.70926 \AA, \lambda_{2}=0.71354$ $\AA$ ), using a $1^{\circ}$ takeoff angle and a $0.02^{\circ}$ slit. These values were used in a least-squares refinement to deter-
mine the lattice parameters. Three-dimensional intensity data to $20=50^{\circ}$ were collected by the stationarycrystal, stationary-counter method, using Zr-filtered Mo $K \alpha$ radiation and a $5^{\circ}$ takeoff angle. Each peak was counted for 10 s , and the background (at $2^{\circ}$ below the $2 \theta$ value of the peak) was counted for 10 s . In total, 3930 reflections were measured. Of these, 1472 were considered observed on the basis that the peak measurement exceeded the background by 4 counts. Only the observed reflections were used for structure analyses and refinements. Lorentz, polarization and $\varphi$ angle absorption corrections with variation less than $20 \%$ were applied. The linear absorption coefficient is $34 \mathrm{~cm}^{-1}$ with $\varrho_{\text {cal }}=3.67 \mathrm{~g} \mathrm{~cm}^{-3}$. The variance for the structure factors was calculated from

$$
\sigma(F)=\frac{1}{2}\left[K \frac{1+I_{B} / I_{P}}{1-I_{B} / I_{P}}\right]^{1 / 2}
$$

where $I_{B}$ is the background count, $I_{P}$ the peak count, and $K$ the product of the Lorentz, polarization, and absorption corrections.

Table 1. Final atomic parameters for $\mathrm{YbP}_{3} \mathrm{O}_{9}$ Space group: $P 2_{1} / c, Z=12$
Cell dimensions: $a=11.219$ (2), $b=19.983$ (3), $c=9.999$ (3) $\AA$, $\beta=97 \cdot 30(2)^{\circ}$

|  | $x$ | $y$ | $z$ | $B(\AA)$ |
| :---: | :---: | :---: | :---: | :---: |
| Yb (1) | 0 | 0 | 0 | 0.44 (9) |
| Yb (2) | $\frac{1}{2}$ | 0 | 0 | $0 \cdot 90$ (9) |
| $\mathrm{Yb}(3)$ | $0 \cdot 0055$ (3) | $0 \cdot 1753$ (1) | 0.4778 (3) | 0.58 (7) |
| $\mathrm{Yb}(4)$ | $0 \cdot 5011$ (3) | $0 \cdot 6570$ (1) | 0.0134 (3) | $0 \cdot 62$ (8) |
| $\mathrm{P}(1)$ | $0 \cdot 248$ (2) | $0 \cdot 1091$ (7) | 0.0256 (2) | $0 \cdot 3$ (3) |
| $\mathrm{P}(2)$ | 0.369 (2) | 0.2197 (8) | 0.179 (2) | $0 \cdot 3$ (3) |
| $\mathrm{P}(3)$ | $0 \cdot 178$ (2) | $0 \cdot 290$ (1) | 0.310 (2) | $1 \cdot 2$ (4) |
| P(4) | 0.226 (2) | 0.447 (1) | 0.978 (2) | $1 \cdot 6$ (4) |
| $\mathrm{P}(5)$ | 0.337 (2) | 0.539 (1) | 0.187 (2) | $0 \cdot 3$ (3) |
| $\mathrm{P}(6)$ | $0 \cdot 163$ (2) | $0 \cdot 618$ (1) | 0.325 (2) | $1 \cdot 2$ (4) |
| $\mathrm{P}(7)$ | 0.254 (2) | 0.7751 (8) | 0.012 (2) | $0 \cdot 5$ (3) |
| $\mathrm{P}(8)$ | 0.351 (2) | 0.8847 (9) | $0 \cdot 193$ (2) | 0.4 (3) |
| P(9) | $0 \cdot 139$ (2) | 0.937 (1) | $0 \cdot 310$ (2) | $0 \cdot 9$ (4) |
| O(1) | 0.234 (4) | 0.998 (2) | 0.358 (4) | $1 \cdot 0$ (8) |
| O(2) | $0 \cdot 170$ (4) | 0.060 (2) | 0.067 (4) | $2 \cdot 5$ (9) |
| $\mathrm{O}(3)$ | 0.375 (4) | $0 \cdot 087$ (2) | 0.006 (4) | $1 \cdot 1$ (8) |
| O(4) | $0 \cdot 127$ (4) | 0.091 (2) | 0.450 (4) | $0 \cdot 8$ (7) |
| O(5) | 0.353 (4) | 0.088 (2) | 0.453 (4) | $0 \cdot 7$ (7) |
| O(6) | $0 \cdot 253$ (4) | $0 \cdot 170$ (3) | $0 \cdot 126$ (4) | $2 \cdot 5$ (9) |
| O(7) | 0.454 (4) | $0 \cdot 176$ (3) | 0.263 (4) | $2 \cdot 4$ (9) |
| $\mathrm{O}(8)$ | $0 \cdot 115$ (4) | $0 \cdot 249$ (2) | 0.407 (4) | 1.5 (8) |
| O(9) | $0 \cdot 384$ (5) | 0.271 (2) | $0 \cdot 080$ (5) | $3 \cdot 6$ (9) |
| $\mathrm{O}(10)$ | $0 \cdot 277$ (5) | 0.271 (2) | 0.245 (5) | $2 \cdot 9$ (9) |
| O(11) | 0.093 (5) | $0 \cdot 302$ (2) | $0 \cdot 183$ (5) | $2 \cdot 9$ (9) |
| $\mathrm{O}(12)$ | 0.199 (4) | $0 \cdot 357$ (2) | 0.375 (3) | $1 \cdot 1$ (6) |
| O(13) | $0 \cdot 242$ (4) | $0 \cdot 479$ (2) | $0 \cdot 118$ (4) | 1.0 (8) |
| O(14) | 0.404 (4) | 0.518 (2) | $0 \cdot 307$ (4) | 0.9 (8) |
| $\mathrm{O}(15)$ | 0.355 (4) | $0 \cdot 592$ (2) | 0.094 (4) | $1 \cdot 3$ (8) |
| $\mathrm{O}(16)$ | 0.099 (3) | 0.586 (2) | 0.432 (3) | 0.6 (6) |
| O(17) | 0.216 (4) | 0.591 (2) | $0 \cdot 210$ (4) | $2 \cdot 2$ (8) |
| O (18) | 0.055 (4) | 0.657 (2) | 0.223 (4) | 0.9 (8) |
| O(19) | $0 \cdot 248$ (4) | 0.667 (2) | 0.392 (5) | 0.9 (8) |
| $\mathrm{O}(20)$ | $0 \cdot 137$ (5) | 0.756 (3) | 0.031 (5) | $2 \cdot 9$ (9) |
| $\mathrm{O}(21)$ | 0.354 (6) | $0 \cdot 819$ (3) | $0 \cdot 119$ (6) | $4 \cdot 0$ (9) |
| $\mathrm{O}(22)$ | $0 \cdot 345$ (4) | $0 \cdot 780$ (2) | $0 \cdot 473$ (4) | $1 \cdot 3$ (7) |
| $\mathrm{O}(23)$ | $0 \cdot 439$ (4) | 0.874 (2) | 0.315 (4) | 2.0 (9) |
| O (24) | $0 \cdot 102$ (3) | 0.905 (2) | $0 \cdot 428$ (3) | $0 \cdot 6$ (6) |
| $\mathrm{O}(25)$ | 0.209 (4) | $0 \cdot 893$ (2) | $0 \cdot 225$ (4) | 1.7 (8) |
| $\mathrm{O}(26)$ | $0 \cdot 362$ (4) | 0.941 (2) | 0.099 (4) | $2 \cdot 1$ (9) |
| $\mathrm{O}(27)$ | $0 \cdot 048$ (4) | $0 \cdot 964$ (2) | $0 \cdot 212$ (4) | $0 \cdot 7$ (6) |

Table 2. Bond distances $(\AA)$ and angles $\left({ }^{\circ}\right)$ for $\mathrm{YbP}_{3} \mathrm{O}_{9}$

| Octahedron around $\mathrm{Yb}(1)$ |  |  |  |  |
| :--- | :---: | :--- | ---: | :---: |
| $\mathrm{Yb}(1)-\mathrm{O}(2)$ | $2 \times 22.28(4)$ | $\mathrm{O}(2)-\mathrm{Yb}(1)-\mathrm{O}(16)$ | $86(1)$ |  |
| $\mathrm{Yb}(1)-\mathrm{O}(16)$ | $2 \times 2 \cdot 20(4)$ | $\mathrm{O}(2)-\mathrm{Yb}(1)-\mathrm{O}(16)$ | $94(1)$ |  |
| $\mathrm{Yb}(1)-\mathrm{O}(27)$ | $2 \times 2 \cdot 23(4)$ | $\mathrm{O}(2)-\mathrm{Yb}(1)-\mathrm{O}(27)$ | $102(1)$ |  |
| $\mathrm{O}(2)-\mathrm{Yb}(1)-\mathrm{O}(2)$ | 180 | $\mathrm{O}(2)-\mathrm{Yb}(1)-\mathrm{O}(27)$ | $78(1)$ |  |
| $\mathrm{O}(6)-\mathrm{Yb}(1)-\mathrm{O}(16)$ | 180 | $\mathrm{O}(16)-\mathrm{Yb}(1)-\mathrm{O}(27)$ | $92(1)$ |  |
| $\mathrm{O}(27)-\mathrm{Yb}(1)-\mathrm{O}(27)$ | 180 | $\mathrm{O}(16)-\mathrm{Yb}(1)-\mathrm{O}(27)$ | $88(1)$ |  |

Octahedron around $\mathrm{Yb}(2)$

| $\mathrm{Yb}(2)-\mathrm{O}(3)$ | $2 \times 2.24(4)$ | $\mathrm{O}(3)-\mathrm{Yb}(2)-\mathrm{O}(14)$ | $95(1)$ |
| :--- | :---: | :---: | :---: |
| $\mathrm{Yb}(2)-\mathrm{O}(14)$ | $2 \times 2.12(4)$ | $\mathrm{O}(3)-\mathrm{Yb}(2)-\mathrm{O}(14)$ | $85(1)$ |
| $\mathrm{Yb}(2)-\mathrm{O}(26)$ | $2 \times 2.27(4)$ | $\mathrm{O}(3)-\mathrm{Yb}(2)-\mathrm{O}(26)$ | $86(1)$ |
| $\mathrm{O}(3)-\mathrm{Yb}(2)-\mathrm{O}(3)$ | 180 | $\mathrm{O}(3)-\mathrm{Yb}(2)-\mathrm{O}(26)$ | $94(1)$ |
| $\mathrm{O}(14)-\mathrm{Yb}(2)-\mathrm{O}(14)$ | 180 | $\mathrm{O}(14)-\mathrm{Yb}(2)-\mathrm{O}(26)$ | $89(1)$ |
| $\mathrm{O}(26)-\mathrm{Yb}(2)-\mathrm{O}(26)$ | 180 | $\mathrm{O}(14)-\mathrm{Yb}(2)-\mathrm{O}(26)$ | $91(1)$ |


| Octahedron around $\mathrm{Yb}(3)$ |  |  |  |  |
| :--- | :---: | :--- | :--- | :---: |
| $\mathrm{Yb}(3)-\mathrm{O}(4)$ | $2 \cdot 21(4)$ | $\mathrm{O}(4)-\mathrm{Yb}(3)-\mathrm{O}(8)$ | $95(2)$ |  |
| $\mathrm{Yb}(3)-\mathrm{O}(8)$ | $2 \cdot 10(4)$ | $\mathrm{O}(4)-\mathrm{Yb}(3)-\mathrm{O}(11)$ | $94(2)$ |  |
| $\mathrm{Yb}(3)-\mathrm{O}(11)$ | $2 \cdot 2(5)$ | $\mathrm{O}(4)-\mathrm{Yb}(3)-\mathrm{O}(18)$ | $83(2)$ |  |
| $\mathrm{Yb}(3)-\mathrm{O}(18)$ | $2 \cdot 07(4)$ | $\mathrm{O}(4)-\mathrm{Yb}(3)-\mathrm{O}(24)$ | $83(1)$ |  |
| $\mathrm{Yb}(3)-\mathrm{O}(20)$ | $2 \cdot 26(6)$ | $\mathrm{O}(8)-\mathrm{Yb}(3)-\mathrm{O}(11)$ | $88(2)$ |  |
| $\mathrm{Yb}(3)-\mathrm{O}(24)$ | $2 \cdot 28(4)$ | $\mathrm{O}(8)-\mathrm{Yb}(3)-\mathrm{O}(18)$ | $86(2)$ |  |
| $\mathrm{O}(4)-\mathrm{Yb}(3)-\mathrm{O}(20)$ | $169(2)$ | $\mathrm{O}(8)-\mathrm{Yb}(3)-\mathrm{O}(20)$ | $86(2)$ |  |
| $\mathrm{O}(8)-\mathrm{Yb}(3)-\mathrm{O}(18)$ | $175(2)$ | $\mathrm{O}(11)-\mathrm{Yb}(3)-\mathrm{O}(20)$ | $97(2)$ |  |
| $\mathrm{O}(11)-\mathrm{Yb}(3)-\mathrm{O}(18)$ | $172(2)$ | $\mathrm{O}(11)-\mathrm{Yb}(3)-\mathrm{O}(24)$ | $88(1)$ |  |
|  |  | $\mathrm{O}(18)-\mathrm{Yb}(3)-\mathrm{O}(20)$ | $87(2)$ |  |
|  |  | $\mathrm{O}(18)-\mathrm{Yb}(3)-\mathrm{O}(24)$ | $99(1)$ |  |
|  | $\mathrm{O}(20)-\mathrm{Yb}(3)-\mathrm{O}(24)$ | $96(2)$ |  |  |


|  |  |  |  |
| :--- | :---: | :--- | :--- |
| $\mathrm{Yb}(4)-\mathrm{O}(5)$ | $2 \cdot 13(4)$ | $\mathrm{O}(5)-\mathrm{Yb}(4)-\mathrm{O}(7)$ | $83(2)$ |
| $\mathrm{Yb}(4)-\mathrm{O}(7)$ | $2 \cdot 26(4)$ | $\mathrm{O}(5)-\mathrm{Yb}(4)-\mathrm{O}(9)$ | $91(2)$ |
| $\mathrm{Yb}(4)-\mathrm{O}(9)$ | $2 \cdot 22(5)$ | $\mathrm{O}(5)-\mathrm{Yb}(4)-\mathrm{O}(15)$ | $98(2)$ |
| $\mathrm{Yb}(4)-\mathrm{O}(15)$ | $2 \cdot 31(4)$ | $\mathrm{O}(5)-\mathrm{Yb}(4)-\mathrm{O}(23)$ | $97(2)$ |
| $\mathrm{Yb}(4)-\mathrm{O}(22)$ | $2 \cdot 15(4)$ | $\mathrm{O}(7)-\mathrm{Yb}(4)-\mathrm{O}(9)$ | $104(2)$ |
| $\mathrm{Yb}(4)-\mathrm{O}(23)$ | $2 \cdot 11(4)$ | $\mathrm{O}(7)-\mathrm{Yb}(4)-\mathrm{O}(15)$ | $80(2)$ |
| $\mathrm{O}(5)-\mathrm{Yb}(4)-\mathrm{O}(22)$ | $175(2)$ | $\mathrm{O}(7)-\mathrm{Yb}(4)-\mathrm{O}(22)$ | $99(2)$ |
| $\mathrm{O}(7)-\mathrm{Yb}(4)-\mathrm{O}(23)$ | $170(2)$ | $\mathrm{O}(9)-\mathrm{Yb}(4)-\mathrm{O}(22)$ | $93(2)$ |
| $\mathrm{O}(9)-\mathrm{Yb}(4)-\mathrm{O}(15)$ | $171(2)$ | $\mathrm{O}(9)-\mathrm{Yb}(4)-\mathrm{O}(23)$ | $86(2)$ |
|  |  | $\mathrm{O}(15)-\mathrm{Yb}(4)-\mathrm{O}(22)$ | $78(2)$ |
|  | $\mathrm{O}(15)-\mathrm{Yb}(4)-\mathrm{O}(23)$ | $90(2)$ |  |
|  | $\mathrm{O}(22)-\mathrm{Yb}(4)-\mathrm{O}(23)$ | $80(2)$ |  |


|  | Tet | on around |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | around |  | Angles of O-P-O |
| $\mathrm{P}(1)-\mathrm{O}(2)$ | $1 \cdot 41$ (4) | $\mathrm{O}(2)-\mathrm{O}(3)$ | $2 \cdot 51$ (6) | 117 (3) |
| $\mathrm{P}(1)-\mathrm{O}(3)$ | 1.53 (5) | $\mathrm{O}(2)-\mathrm{O}(6)$ | $2 \cdot 44$ (6) | 109 (3) |
| $\mathrm{P}(1)-\mathrm{O}(6)$ | 1.57 (5) | $\mathrm{O}(2)-\mathrm{O}(12)$ | $2 \cdot 58$ (6) | 113 (3) |
| $\mathrm{P}(1)-\mathrm{O}(12)$ | $1 \cdot 68$ (3) | $\mathrm{O}(3)-\mathrm{O}(6)$ | 2.52 (6) | 110 (3) |
|  |  | $\mathrm{O}(3)-\mathrm{O}(12)$ | 2.47 (6) | 102 (2) |
|  |  | $\mathrm{O}(6)-\mathrm{O}(12)$ | $2 \cdot 56$ (6) | 104 (2) |

Tetrahedron around $\mathbf{P}(\mathbf{2})$

| $\mathrm{P}(2)-\mathrm{O}(6)$ | $1.67(5)$ | $\mathrm{O}(6)-\mathrm{O}(7)$ | $2.47(6)$ | $104(3)$ |
| :--- | :--- | :--- | :--- | ---: |
| $\mathrm{P}(2)-\mathrm{O}(7)$ | $1.47(5)$ | $\mathrm{O}(6)-\mathrm{O}(9)$ | $2.55(6)$ | $111(3)$ |
| $\mathrm{P}(2)-\mathrm{O}(\mathrm{P})$ | $1.45(5)$ | $\mathrm{O}(6)-\mathrm{O}(10)$ | $2.34(6)$ | $90(3)$ |
| $\mathrm{P}(2)-\mathrm{O}(10)$ | $1.65(5)$ | $\mathrm{O}(7)-\mathrm{O}(9)$ | $2.68(6)$ | $133(3)$ |
|  |  | $\mathrm{O}(7)-\mathrm{O}(10)$ | $2.72(6)$ | $122(3)$ |
|  |  | $\mathrm{O}(9)-\mathrm{O}(10)$ | $2.16(6)$ | $88(3)$ |


| $\mathrm{P}(3)-\mathrm{O}(8)$ | $1.51(4)$ | $\mathrm{O}(8)-\mathrm{O}(10)$ | $2.62(6)$ | $128(3)$ |
| :--- | :--- | :--- | :--- | ---: |
| $\mathrm{P}(3)-\mathrm{O}(10)$ | $1.41(6)$ | $\mathrm{O}(8)-\mathrm{O}(11)$ | $2.48(6)$ | $109(3)$ |
| $\mathrm{P}(3)-\mathrm{O}(11)$ | $1.51(6)$ | $\mathrm{O}(8)-\mathrm{O}(12)$ | $2.40(6)$ | $106(2)$ |
| $\mathrm{P}(3)-\mathrm{O}(12)$ | $1.49(4)$ | $\mathrm{O}(10)-\mathrm{O}(1)$ | $2.18(6)$ | $96(3)$ |
|  |  | $\mathrm{O}(10)-\mathrm{O}(12)$ | $2.38(6)$ | $111(3)$ |
|  |  | $\mathrm{O}(11)-\mathrm{O}(12)$ | $2.41(6)$ | $106(2)$ |

Table 2 (cont.)
Tetrahedron around $P(4)$

| $\mathrm{P}(4)-\mathrm{O}(1)$ | $1.64(4)$ | $\mathrm{O}(1)-\mathrm{O}(4)$ | $2.46(6)$ | $110(3)$ |
| :--- | :--- | :--- | :--- | ---: |
| $\mathrm{P}(4)-\mathrm{O}(4)$ | $1.35(5)$ | $\mathrm{O}(1)-\mathrm{O}(5)$ | $2.36(6)$ | $92(2)$ |
| $\mathrm{P}(4)-\mathrm{O}(5)$ | $1.63(5)$ | $\mathrm{O}(1) \mathrm{O}(13)$ | $2.63(6)$ | $112(2)$ |
| $\mathrm{P}(4)-\mathrm{O}(13)$ | $1.53(4)$ | $\mathrm{O}(4)-\mathrm{O}(5)$ | $2.53(6)$ | $116(3)$ |
|  |  | $\mathrm{O}(4)-\mathrm{O}(13)$ | $2.42(6)$ | $115(3)$ |
|  |  | $\mathrm{O}(5)-\mathrm{O}(13)$ | $2.56(6)$ | $108(3)$ |

Tetrahedron around $\mathrm{P}(5)$

| $\mathrm{P}(5)-\mathrm{O}(13)$ | $1.69(5)$ | $\mathrm{O}(13)-\mathrm{O}(14)$ | $2 \cdot 57(6)$ | $112(3)$ |
| :--- | :--- | :--- | :--- | ---: |
| $\mathrm{P}(5)-\mathrm{O}(14)$ | $1.40(5)$ | $\mathrm{O}(13)-\mathrm{O}(15)$ | $2 \cdot 61(6)$ | $113(2)$ |
| $\mathrm{P}(5)-\mathrm{O}(15)$ | $1.44(4)$ | $\mathrm{O}(13)-\mathrm{O}(17)$ | $2 \cdot 47(6)$ | $91(2)$ |
| $\mathrm{P}(5)-\mathrm{O}(17)$ | $1.75(5)$ | $\mathrm{O}(14)-\mathrm{O}(15)$ | $2 \cdot 59(6)$ | $132(3)$ |
|  |  | $\mathrm{O}(14)-\mathrm{O}(17)$ | $2 \cdot 65(6)$ | $114(3)$ |
|  |  | $\mathrm{O}(15)-\mathrm{O}(17)$ | $2 \cdot 06(6)$ | $80(2)$ |


| Tetrahedron around $\mathrm{P}(6)$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Angles of $\mathrm{O}-\mathrm{P}-\mathrm{O}$ |
| $\mathrm{P}(6)-\mathrm{O}(16)$ | $1 \cdot 50$ (4) | $\mathrm{O}(16)-\mathrm{O}(17)$ | $2 \cdot 72$ (6) | 133 (3) |
| $\mathrm{P}(6)-\mathrm{O}(17)$ | $1 \cdot 46$ (4) | $\mathrm{O}(16)-\mathrm{O}(18)$ | $2 \cdot 53$ (6) | 105 (2) |
| $\mathrm{P}(6)-\mathrm{O}(18)$ | $1 \cdot 67$ (5) | $\mathrm{O}(16)-\mathrm{O}(19)$ | $2 \cdot 39$ (6) | 107 (3) |
| $\mathrm{P}(6)-\mathrm{O}(19)$ | $1 \cdot 47$ (5) | $\mathrm{O}(17)-\mathrm{O}(18)$ | $2 \cdot 25$ (6) | 91 (2) |
|  |  | $\mathrm{O}(17)-\mathrm{O}(19)$ | $2 \cdot 35$ (6) | 107 (3) |
|  |  | $\mathrm{O}(18)-\mathrm{O}(19)$ | $2 \cdot 60$ (6) | 110 (3) |
| Tetrahedron around $\mathrm{P}(7)$ |  |  |  |  |
| $\mathrm{P}(7)-\mathrm{O}(19)$ | 1.66 (5) | $\mathrm{O}(19)-\mathrm{O}(20)$ | $2 \cdot 52$ (6) | 110 (3) |
| $\mathrm{P}(7)-\mathrm{O}(20)$ | $1 \cdot 40$ (6) | $\mathrm{O}(19)-\mathrm{O}(21)$ | $2 \cdot 45$ (6) | 93 (3) |
| $\mathrm{P}(7)-\mathrm{O}(21)$ | $1 \cdot 69$ (6) | $\mathrm{O}(19)-\mathrm{O}(22)$ | $2 \cdot 60$ (6) | 106 (2) |
| $\mathrm{P}(7)-\mathrm{O}(22)$ | $1 \cdot 58$ (5) | $\mathrm{O}(20)-\mathrm{O}(21)$ | $2 \cdot 82$ (6) | 128 (3) |
|  |  | $\mathrm{O}(20)-\mathrm{O}(22)$ | $2 \cdot 59$ (6) | 119 (3) |
|  |  | $\mathrm{O}(21)-\mathrm{O}(22)$ | $2 \cdot 47$ (6) | 97 (3) |

Tetrahedron around $\mathrm{P}(8)$

| $\mathrm{P}(8)-\mathrm{O}(21)$ | $1 \cdot 51(6)$ | $\mathrm{O}(21)-\mathrm{O}(23)$ | $2 \cdot 30(6)$ | $103(3)$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(8)-\mathrm{O}(23)$ | $1 \cdot 48(5)$ | $\mathrm{O}(21)-\mathrm{O}(25)$ | $2 \cdot 53(6)$ | $105(3)$ |
| $\mathrm{P}(8)-\mathrm{O}(25)$ | $1.67(5)$ | $\mathrm{O}(21)-\mathrm{O}(26)$ | $2 \cdot 44(6)$ | $110(3)$ |
| $\mathrm{P}(8)-\mathrm{O}(26)$ | $1.48(4)$ | $\mathrm{O}(23)-\mathrm{O}(25)$ | $2 \cdot 63(6)$ | $114(3)$ |
|  |  | $\mathrm{O}(23)-\mathrm{O}(26)$ | $2 \cdot 60(6)$ | $122(3)$ |
|  |  | $\mathrm{O}(25)-\mathrm{O}(26)$ | $2 \cdot 45(6)$ | $102(2)$ |

Tetrahedron around $P(9)$

| $\mathrm{P}(9)-\mathrm{O}(1)$ | $1.65(5)$ | $\mathrm{O}(1)-\mathrm{O}(24)$ | $2 \cdot 54(6)$ | $109(2)$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(9)-\mathrm{O}(24)$ | $1.45(4)$ | $\mathrm{O}(1)-\mathrm{O}(25)$ | $2.48(6)$ | $103(3)$ |
| $\mathrm{P}(9)-\mathrm{O}(25)$ | $1.51(5)$ | $\mathrm{O}(1)-\mathrm{O}(27)$ | $2.50(6)$ | $107(3)$ |
| $\mathrm{P}(9)-\mathrm{O}(27)$ | $1.43(5)$ | $\mathrm{O}(24)-\mathrm{O}(25)$ | $2.51(6)$ | $115(3)$ |
|  |  | $\mathrm{O}(24)-\mathrm{O}(27)$ | $2.47(6)$ | $118(3)$ |
|  |  | $\mathrm{O}(25)-\mathrm{O}(27)$ | $2.30(6)$ | $102(3)$ |

Inter-tetrahedral angles

| $\mathrm{P}(4)-\mathrm{O}(1)--\mathrm{P}(9)$ | $128(3)$ | $\mathrm{P}(5)-\mathrm{O}(17)-\mathrm{P}(6)$ | $136(3)$ |
| :--- | :---: | :--- | :--- |
| $\mathrm{P}(1)-\mathrm{O}(6)--\mathrm{P}(2)$ | $129(3)$ | $\mathrm{P}(6)-\mathrm{O}(19)-\mathrm{P}(7)$ | $139(3)$ |
| $\mathrm{P}(2)-\mathrm{O}(10)-\mathrm{P}(3)$ | $157(4)$ | $\mathrm{P}(7)-\mathrm{O}(21)-\mathrm{P}(8)$ | $135(5)$ |
| $\mathrm{P}(1)-\mathrm{O}(12)-\mathrm{P}(3)$ | $140(3)$ | $\mathrm{P}(8)-\mathrm{O}(25)-\mathrm{P}(9)$ | $138(3)$ |
| $\mathrm{P}(4)-\mathrm{O}(13)-\mathrm{P}(5)$ | $132(3)$ |  |  |

## Structure determination

The heavy-atom method was used to solve the structure. It was observed that the reflections with $k=3 n$ were stronger than the others. This indicates that heavy atoms have a repeat distance of one-third of the $b$ axis. A three-dimensional Patterson map showed distinct strong interactions at $\left(\frac{1}{2}, 0,0\right),\left(0, \frac{1}{6}, \frac{1}{2}\right),\left(\frac{1}{2}, \frac{1}{6}, \frac{1}{2}\right)$, $\left(0, \frac{1}{3}, 0\right),\left(\frac{1}{2}, \frac{1}{3}, 0\right),\left(0, \frac{1}{2}, \frac{1}{2}\right)$, and $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, from which it was concluded that 12 Yb atoms are located in four independent positions, at $2(a), 2(b)$ and two sets of

4(e) in space group $P 2_{1} / c$. A structure-factor calculation based on these positions gave a value of 0.280 for the difference function $R=\sum\left|F_{o}-F_{c}\right| / \sum\left|F_{o}\right|$. Published scattering factors for $\mathrm{Yb}^{3+}, \mathrm{P}$ and O (Cromer

Table 3. Structure factors
\& Waber, 1965) and anomalous dispersion coefficients for Mo radiation (Cromer, 1965) were used in this and subsequent calculations. A difference Fourier map based on this model did not give a reasonable P atom arrangement. Alternative models were constructed, all taking account of the fact that in metaphosphates $(\mathrm{O} / \mathrm{P}=3)$ the $\mathrm{PO}_{4}$ tetrahedra share corners to form onedimensional chains. For the best of these models, with nine independent P atoms, $R$ was reduced to $0 \cdot 178$. A subsequent difference Fourier map clearly revealed 27 independent O positions. The atomic parameters and
isotropic temperature factors were then refined, using a full-matrix least-squares program (Prewitt, 1962). Their final values are listed in Table 1, and the bond distances and angles are listed in Table 2. Table 3 gives the observed and calculated structure factors. For all observed reflections, $R=0.076$ and the weighted $R_{w}=$ 0.064 , where $R_{w}=\left[\sum w\left(F_{o}-F_{c}\right)^{2} / \sum w F_{o}^{2}\right]^{1 / 2}$ with $w=1 / \sigma^{2}$. In the final cycle the maximum and average shift/error are 0.61 and 0.11 , respectively. No correction was made for secondary extinction. A final difference synthesis revealed no physically significant peaks.


Fig. 1. An $a b$ projection of the arrangement of $\mathrm{PO}_{4}$ tetrahedra and $\mathrm{YbO}_{6}$ octahedra in $\mathrm{YbP}_{3} \mathrm{O}_{9}$. The octahedra are slightly distorted.


Fig. 2. A bc projection of the arrangement of $\mathrm{PO}_{4}$ tetrahedra in $\mathrm{YbP}_{3} \mathrm{O}_{9}$, showing the ribbons that run helically along the $c$ axis.

## Discussion

The structure of $\mathrm{YbP}_{3} \mathrm{O}_{9}$ is illustrated in Figs. 1 and 2. The basic structural units are helical ribbons, $\left(\mathrm{PO}_{3}\right)_{n}$, formed by corner-sharing of $\mathrm{PO}_{4}$ tetrahedra. The ribbons run along the $c$ axis and are joined to each other by $\mathrm{Yb}-\mathrm{O}$ bonds. Each Yb atom is coordinated with six O atoms to form slightly distorted octahedra that are isolated from each other. The shortest $\mathrm{Yb}-\mathrm{Yb}$ distance is $5.610 \AA$. Each O atom is bonded to two cations, either two P atoms or one P and one Yb atom; no O atom is common to two Yb atoms. In contrast, in $\mathrm{NdP}_{3} \mathrm{O}_{9}$ every Nd atom shares two O atoms with a second Nd atom. In that structure, each Nd atom is coordinated to eight O atoms. Most $\mathrm{P}-\mathrm{O}$ distances are longer for O atoms bonded to two P atoms than for those bonded to one P and one Yb atom, probably because of the repulsion between $\mathrm{P}^{5+}$ ions. Longer bond distances in P-O-P bonding were also observed in $\mathrm{NdP}_{3} \mathrm{O}_{9}, \mathrm{NdP}_{5} \mathrm{O}_{14}$ (Hong, 1974) and $\mathrm{YbP}_{5} \mathrm{O}_{14}$ (Hong \& Pierce, 1974). Due to ionic repulsion, $\mathrm{P}^{5+}$ ions do not form edge-shared tetrahedra, although they can exhibit corner-sharing. Because of their increased charge, $\mathrm{Mo}^{6+}, \mathrm{W}^{6+}$ or $\mathrm{Re}^{7+}$ can form only isolated tetrahedra.

The fact that the $\mathrm{Nd}^{3+}$ fluorescence lifetime is considerably longer in $\mathrm{Y}_{0.9} \mathrm{Nd}_{0.1} \mathrm{P}_{3} \mathrm{O}_{9}(490 \mu \mathrm{~s})$ than in $\mathrm{La}_{0.9} \mathrm{Nd}_{0.1} \mathrm{P}_{3} \mathrm{O}_{9}(160 \mu \mathrm{~s})$ can be attributed in part to the reduction in $\mathrm{Nd}-\mathrm{Nd}$ pair interactions and therefore to reduction of concentration quenching in the $\mathrm{YbP}_{3} \mathrm{O}_{9}$ structure because of the isolation of the rare-earth ions from each other. The difference also results in part because the limiting lifetime for very dilute solutions of $\mathrm{Nd}^{3+}$ in $\mathrm{YP}_{3} \mathrm{O}_{9}$ is at least $490 \mu \mathrm{~s}$, compared with 375 and $325 \mu$ s for such solutions in $\mathrm{LaP}_{3} \mathrm{O}_{9}$ and $\mathrm{LaP}_{5} \mathrm{O}_{14}$, respectively (Dwight, Hong \& Pierce, 1973).

The observation of a particularly high lifetime in $\mathrm{Y}_{0.9} \mathrm{Nd}_{0.1} \mathrm{P}_{3} \mathrm{O}_{9}$ makes the $\mathrm{Y}_{1-x} \mathrm{Nd}_{x} \mathrm{P}_{3} \mathrm{O}_{9}$ solid solution a source of promising materials for efficient $\mathrm{Nd}^{3+}$ lasers. Another potential advantage is that the concentration of rare-earth ions in the $\mathrm{YbP}_{3} \mathrm{O}_{9}$ structure is $5.4 \times 10^{21} \mathrm{~cm}^{-3}$, compared with only $3.9 \times 10^{21} \mathrm{~cm}^{-3}$ in $\mathrm{La}_{1-x} \mathrm{Nd}_{x} \mathrm{P}_{5} \mathrm{O}_{14}$ solid solutions, which have already been used for the fabrication of lasers. However, since $\mathrm{YP}_{3} \mathrm{O}_{9}$ and $\mathrm{NdP}_{3} \mathrm{O}_{9}$ have different structures, the solubility of $\mathrm{Nd}^{+3}$ in $\mathrm{YP}_{3} \mathrm{O}_{9}$ must be limited. The maximum value of the solubility was observed about $x=0.20$ in the series of $\mathrm{Y}_{1-x} \mathrm{Nd}_{x} \mathrm{P}_{3} \mathrm{O}_{9}$.

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# The Crystal Structure of t-Amyloxycarbonyl-L-prolyl-L-prolyl-L-proline 

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The crystal is orthorhombic, space group $P 2_{1} 2_{1} 2_{1}$, with $a=14 \cdot 315, b=9 \cdot 924, c=15 \cdot 916 \AA, Z=4.1897$ non-zero reflexions collected on a diffractometer $(\sin \theta / \lambda=0.575)$ were used in the structure determination. The structure was solved by the symbolic addition method and refined by the least-squares method to an $R$ of 0.06 . The steric hindrance between pyrrolidine rings of nearest neighbours significantly affects the bond angles of the peptide linkages in order to suppress the effect of overcrowding. One of the $\mathrm{C}^{\gamma}$ atoms of a pyrrolidine ring shows remarkable anomalous behaviour. The peptide bonds are trans, and the conformation of the peptide chains is fairly close to those of poly-L-proline II.

## Introduction

X-ray studies on a group of synthetic peptides, which are related to collagen as collagenase substrates or as

[^1]collagen model polypeptides, have been carried out as one of the main research projects in this laboratory. The present study of the crystal structure of t -amyloxy-carbonyl-L-prolyl-L-prolyl-L-proline is part of this project. Conformational studies of proline oligomers carried out recently by spectroscopic methods revealed the following structural characteristics (Isemura,


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